HYDRAULIC TURBOMACHINES
Exercises 2
Velocity Triangles

Parametric Study for a Velocity Triangle of a Francis Turbine

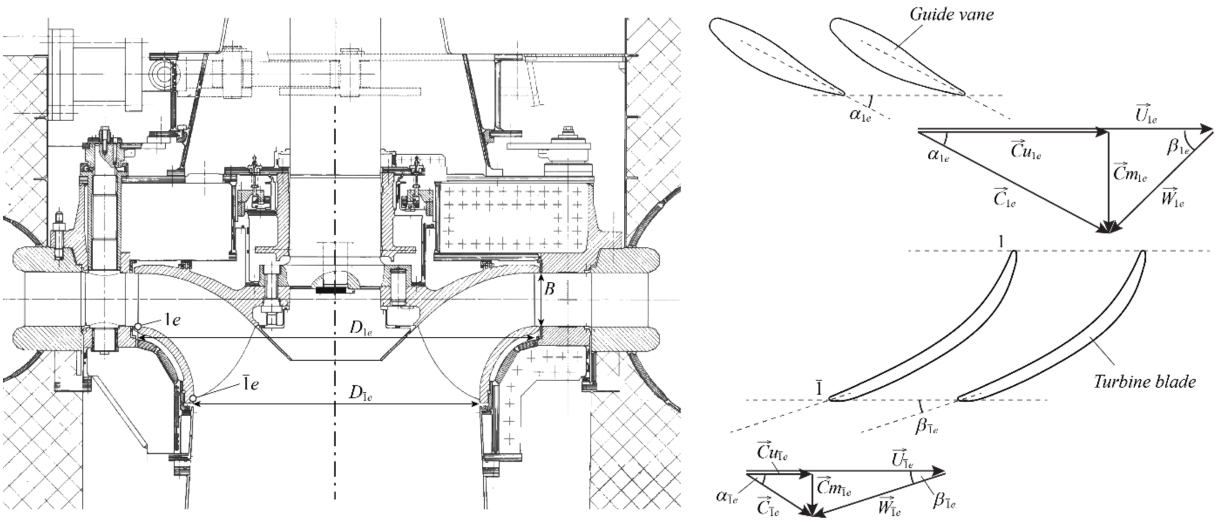

Figure 1. Scheme of the hydropower plant with its characteristics.

The meridional view of a Francis turbine and one example of the velocity triangle are sketched in Figure 1. For a Francis turbine, the angle of the absolute flow velocity at the inlet α_{1e} corresponds to the guide vane opening degree, and the angle of the relative flow velocity at the outlet corresponds to the outlet blade angle $\beta_{\bar{1}e}$, as shown in Figure 1. Referring to the figure, answer the following questions.

1) Give the expression of turbine rotational velocity U_{1e} and $U_{\bar{1}e}$ as a function of the angular rotation ω and the inlet and outlet diameters, D_{1e} and $D_{\bar{1}e}$ respectively.

2) Give the relation of the turbine discharge Q and the discharge Q_t traversing the runner as a function of the volumetric efficiency η_v.

3) Give the meridional components of the flow velocity Cm_{1e} and $Cm_{\bar{1}e}$ as a function of the discharge Q and the volumetric efficiency η_v by using the variable defined in Figure 1.

4) Considering the vectorial relationship at the turbine runner inlet 1, write the relation of π, Cu_{1e}, Q, η_v, D_{1e}, B and α_{1e}.

5) Considering the vectorial relationship at the turbine runner outlet $\bar{1}$, derive the relation of π, $Cu_{\bar{1}e}$, $U_{\bar{1}e}$, Q, η_v, $D_{\bar{1}e}$ and $\beta_{\bar{1}e}$.

Lausanne, 12 October 2023

6) Derive the relation of the transformed specific energy E_t as a function of U_{1e}, $U_{\bar{1}e}$, Q, η_v, D_{1e}, $D_{\bar{1}e}$, B, α_{1e} and $\beta_{\bar{1}e}$.

7) Considering the ratio of $\frac{U_{1e}}{U_{\bar{1}e}}$, derive the relation for transformed specific energy E_t as a function of $U_{\bar{1}e}$, Q, η_v, $D_{\bar{1}e}$, B, α_{1e} and $\beta_{\bar{1}e}$.

8) For a given rotational frequency of the runner, sketch the transformed specific energy E_t as a function of the traversing discharge Q_t, and derive the condition of minimum discharge Q_t^{min} to achieve positive specific energy.

9) When the turbine is operated at the best efficiency point (BEP), express the transformed power P_t by necessary variables, considering the assumption of the best efficiency point $\left(Cu_{\bar{1}e}=0\right)$.

Calculation of the best efficiency using a hill-chart

The $Q_{ED}-n_{ED}$ hill-chart of a Francis turbine with the iso-value curves of both the global efficiency η (red curves) and guide vane opening α (blue curves) is represented in Figure 3. The horizontal and vertical axes represent IEC discharge factor Q_{ED} and IEC speed factor n_{ED}, respectively. Using the hill-chart, answer the following questions. Use the following values if required.

$D_{1e}=4.20$ m, $D_{\bar{1}e}=3.50$ m, $B=0.60$ m, $n=3.88$ Hz, $\eta_v=0.98$ and $\eta_{me}=0.97$

where η_v and η_{me} are the volumetric and mechanical efficiency, respectively.

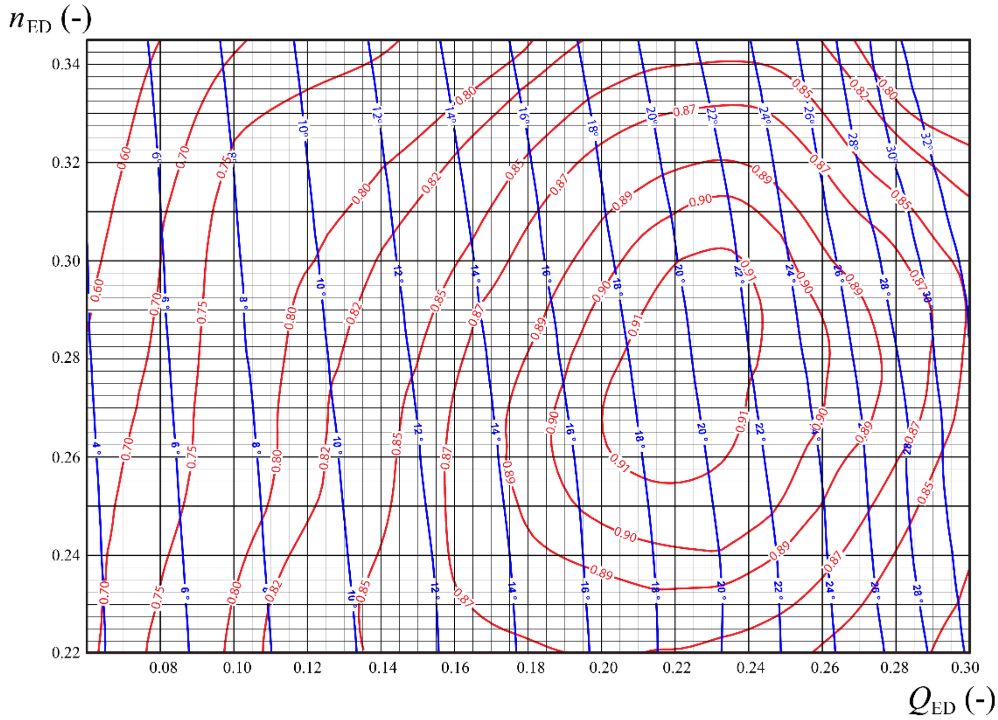

Figure 2. $Q_{ED} - n_{ED}$ *hill-chart of a Francis turbine*

10) Point out the best efficiency point in the hill-chart, and estimate the global efficiency $\eta_{BEP}^{hill-chart}$ and guide vane opening $\alpha_{BEP}^{hill-chart}$ at the best efficiency point (BEP).

11) At the BEP, the available head H and the discharge in the power plant Q are measured as $H = 235$ m and $Q = 130$ m^3 s^{-1}. Calculate the transformed energy E_t at the best efficiency point. Then, calculate the available power at the BEP, i.e. P^{BEP}.